Surds with Algebra

(a) Given that $(3+\sqrt{a})(4+\sqrt{a}) = 17 + b\sqrt{a}$ find the values of a and b.

(b) Given that $(5-\sqrt{x})^2 = y - 10\sqrt{2}$ find the values of x and y.

(c) Given that $(c-\sqrt{d})^2 = 39 - 12\sqrt{d}$, find the values of c and d.

(a) $a=5, b=7$

(b) $x=2, y=27$

(c) $c=6, d=3$

(a) Find the nth term of the sequence
$\sqrt{2} \quad \sqrt{8} \quad \sqrt{18} \quad \sqrt{32}$

(b) Find the nth term of the sequence
$20 \quad 17+\sqrt{5} \quad 14+\sqrt{20} \quad 11+\sqrt{45}$

(a) $\sqrt{2}n$

(b) $(\sqrt{5}-3)n+23-\sqrt{5}$

(a) Show that $\sqrt{25a^2} + \frac{2ab\sqrt{75}}{\sqrt{3b^2}}$ is always a multiple of 5, given that a and b are integers.

(b) Show that $\sqrt{2}(c\sqrt{8} - d\sqrt{50})$ is always even when c and d are integers.

(a) $5a + \frac{2ab \times 5\sqrt{3}}{b\sqrt{3}}$

$= 5a + 10a$

$= 15a = 5(3a)$

(b) $\sqrt{16}c - \sqrt{100}d$

$= 4c - 10d$

$= 2(2c-5d)$

(a) Given that d is a prime number, rationalise the denominator of $\frac{5\sqrt{d}}{2+\sqrt{d}}$

(b) Rationalise the denominator of $\frac{2e+\sqrt{f}}{e-\sqrt{f}}$ where e is an integer and f is a prime number.

(a) $\frac{10\sqrt{d} - 5d}{4-d}$

(b) $\frac{2e^2+f+3e\sqrt{f}}{e^2-f}$

Solve, giving your answer in its simplest form:

(a) $3\sqrt{3} = \sqrt{3}x + 2$

(b) $3x - \sqrt{5} = \sqrt{5}x + 1$

(c) $x^2 - 2\sqrt{3}x + 3 = 0$

(d) $2x^2 + \sqrt{7}x - 7 = 0$

(e) $3x^2 + 5\sqrt{5}x = 10$

(a) $x = \frac{9-2\sqrt{3}}{3}$

(b) $x = 2+\sqrt{5}$

(c) $x = \sqrt{3}$

(d) $x = \frac{\sqrt{7}}{2}$ or $x = -\sqrt{7}$

(e) $x = \frac{\sqrt{5}}{3}$ or $x = -2\sqrt{5}$