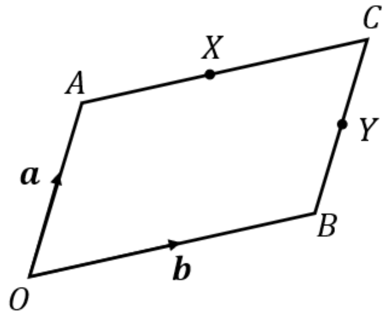
Vector Proof – Parallel Lines

(a)

$OACB$ is a parallelogram. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. X is the midpoint of AC and Y is the midpoint of BC. Show that XY and AB are parallel.



$$\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$$

$$\overrightarrow{XY} = -\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$$

$$\overrightarrow{AB} = 2\overrightarrow{XY}$$

Since $\overrightarrow{AB}$ is a multiple of $\overrightarrow{XY}$, XY is parallel to AB.

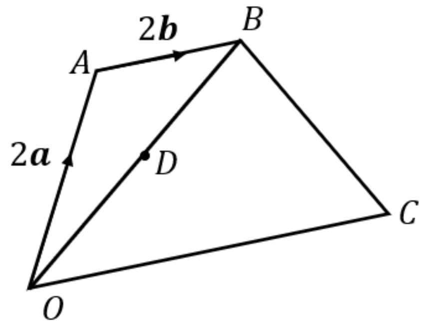
(b)

$OACB$ is a trapezium. $\overrightarrow{OA} = 2\boldsymbol{a}$ and $\overrightarrow{AB} = 2\boldsymbol{b}$. $\overrightarrow{OC} = 2\overrightarrow{AB}$ and D is the midpoint of OB. Show that AD is parallel to BC.

$$\overrightarrow{BC} = -2\boldsymbol{a} + 2\boldsymbol{b}$$

$$\overrightarrow{AD} = -\boldsymbol{a} + \boldsymbol{b}$$

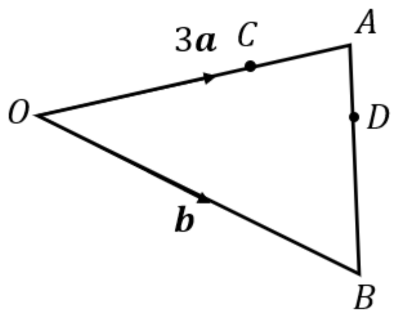
$$\overrightarrow{BC} = 2\overrightarrow{AD}$$



Since $\overrightarrow{BC}$ is a multiple of $\overrightarrow{AD}$, AD is parallel to BC.

(c)

In the triangle OAB, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OA} = 3\boldsymbol{a}$. The point C divides the line OA in the ratio $2:1$ and the point D divides the line AB in the ratio $1:2$. Show that CD is parallel to OB.



$$\overrightarrow{OB} = \boldsymbol{b}$$

$$\overrightarrow{CD} = \boldsymbol{a} + \frac{1}{3}(-3\boldsymbol{a} + \boldsymbol{b}) = \frac{1}{3}\boldsymbol{b}$$

$$\overrightarrow{OB} = 3\overrightarrow{CD}$$

Since $\overrightarrow{OB}$ is a multiple of $\overrightarrow{CD}$, CD is parallel to OB.

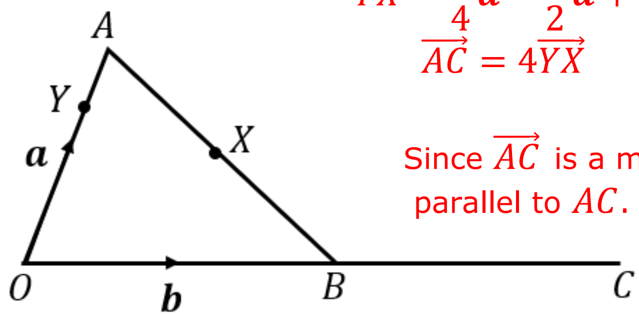
(d)

In the triangle OAB, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OA} = \boldsymbol{a}$. Point B is the midpoint of the line OC and X is the midpoint of AB. The point Y divides the line OA in the ratio $3:1$. Show that YX is parallel to AC.

$$\overrightarrow{AC} = -\boldsymbol{a} + 2\boldsymbol{b}$$

$$\overrightarrow{YX} = \frac{1}{4}\boldsymbol{a} - \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} = -\frac{1}{4}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$$

$$\overrightarrow{AC} = 4\overrightarrow{YX}$$



Since $\overrightarrow{AC}$ is a multiple of $\overrightarrow{YX}$, XY is parallel to AC.